Standard 3—Mathematics
Elementary

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **use models, facts, and relationships to draw conclusions about mathematics and explain their thinking.**
- **use patterns and relationships to analyze mathematical situations.**
- **justify their answers and solution processes.**
- **use logical reasoning to reach simple conclusions.**

This is evident, for example, when students:

▲ build geometric figures out of straws.
▲ find patterns in sequences of numbers, such as the triangular numbers 1, 3, 6, 10, ...
▲ explore number relationships with a calculator (e.g., 12 + 6 = 18, 11 + 7 = 18, etc.) and draw conclusions.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.**
- **use concrete materials to model numbers and number relationships for whole numbers and common fractions, including decimal fractions.**
- **relate counting to grouping and to place-value.**
- **recognize the order of whole numbers and commonly used fractions and decimals.**
- **demonstrate the concept of percent through problems related to actual situations.**

This is evident, for example, when students:

▲ count out 15 small cubes and exchange ten of the cubes for a rod ten cubes long.
▲ use the number line to show the position of 1/4.
▲ figure the tax on $4.00 knowing that taxes are 7 cents per $1.00.

Sample Problems

16. Marlene is designing a uniform for her soccer team. She can choose from 2 different shirts and 3 different pairs of shorts. How many different uniforms can she make if she uses all the shirts and all the shorts?

Answer _____

Explain how you got your answer with a picture or diagram.

Ms. Rivera's class must collect 180 soda cans to win the recycling contest. The chart below shows how the class is doing. How many cans must they collect in the fourth week to reach the goal of 180?

Week	Cans
1	42
2	74
3	18
4	
Goal	180

Answer _____

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **add, subtract, multiply, and divide whole numbers.**
- **develop strategies for selecting the appropriate computational and operational method in problem-solving situations.**
- **know single digit addition, subtraction, multiplication, and division facts.**
- **understand the commutative and associative properties.**

This is evident, for example, when students:

- ▲ use the fact that multiplication is commutative (e.g., 2 x 7 = 7 x 2), to assist them with their memorizing of the basic facts.
- ▲ solve multiple-step problems that require at least two different operations.
- ▲ progress from base ten blocks to concrete models and then to paper and pencil algorithms.

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **use concrete materials to model spatial relationships.**
- **construct tables, charts, and graphs to display and analyze real-world data.**
- **use multiple representations (simulations, manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures.**
- **use variables such as height, weight, and hand size to predict changes over time.**
- **use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.**

This is evident, for example, when students:

- ▲ build a 3 x 3 x 3 cube out of blocks.
- ▲ use square tiles to model various rectangles with an area of 24 square units.
- ▲ read a bar graph of population trends and write an explanation of the information it contains.

Sample Problems

7. Shanelle earns $3.50 per hour for babysitting. Each week she babysits for 4 hours.

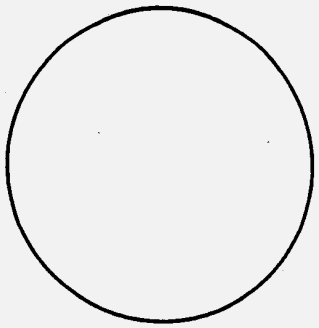
A) How much money does she earn in 1 week?

Answer _____

B) How much money does she earn in 4 weeks?

Answer _____

11. Bobbie's family bought a pizza. Her mother and sister together ate $\frac{1}{2}$ of the pizza. Bobbie ate $\frac{1}{2}$ of what was left. Use the circle to draw a picture that shows how much of the pizza Bobbi ate.



What fraction of the whole pizza did Bobbie eat?

Answer ☐

Standard 3—Mathematics
Elementary

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:
- **understand that measurement is approximate, never exact.**
- **select appropriate standard and nonstandard measurement tools in measurement activities.**
- **understand the attributes of area, length, capacity, weight, volume, time, temperature, and angle.**
- **estimate and find measures such as length, perimeter, area, and volume using both nonstandard and standard units.**
- **collect and display data.**
- **use statistical methods such as graphs, tables, and charts to interpret data.**

This is evident, for example, when students:

▲ measure with paper clips or finger width.
▲ estimate, then calculate, how much paint would be needed to cover one wall.
▲ create a chart to display the results of a survey conducted among the classes in the school, or graph the amounts of survey responses by grade level.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:
- **make estimates to compare to actual results of both formal and informal measurement.**
- **make estimates to compare to actual results of computations.**
- **recognize situations where only an estimate is required.**
- **develop a wide variety of estimation skills and strategies.**
- **determine the reasonableness of results.**
- **predict experimental probabilities.**
- **make predictions using unbiased random samples.**
- **determine probabilities of simple events.**

This is evident, for example, when students:

▲ estimate the length of the room before measuring.
▲ predict the average number of red candies in a bag before opening a group of bags, counting the candies, and then averaging the number that were red.
▲ determine the probability of picking an even numbered slip from a hat containing slips of paper numbered 1, 2, 3, 4, 5, and 6.

Sample Problems

It's Saturday and you're going to meet your friends for lunch and a movie. You have to leave your home at 11:30 AM. Your parents say you can't go until you finish your work. Your work includes your homework and your Saturday chores:

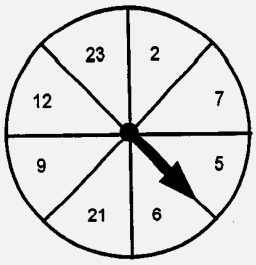
40 minutes of math homework.
30 minutes to clean your room.
15 minutes to fold the laundry
5 minutes to take out the garbage
60 minutes to eat and get ready to go

A) At what time should you get started doing your work? Show all the math you did to figure this out.

Answer _____ AM

B) Describe how you would use your time between when you wake up and when you leave at 11:30 AM to go to lunch and the movie.

The spinner below was used by Jodie's class for the school fair:



A) If the spinner is spun once, what is the probability of the spinner landing on an even number?

Answer ☐

B) If the spinner is spun a second time, what is the probability of the spinner landing on a number that is divisible by 3?

Answer ☐

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Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

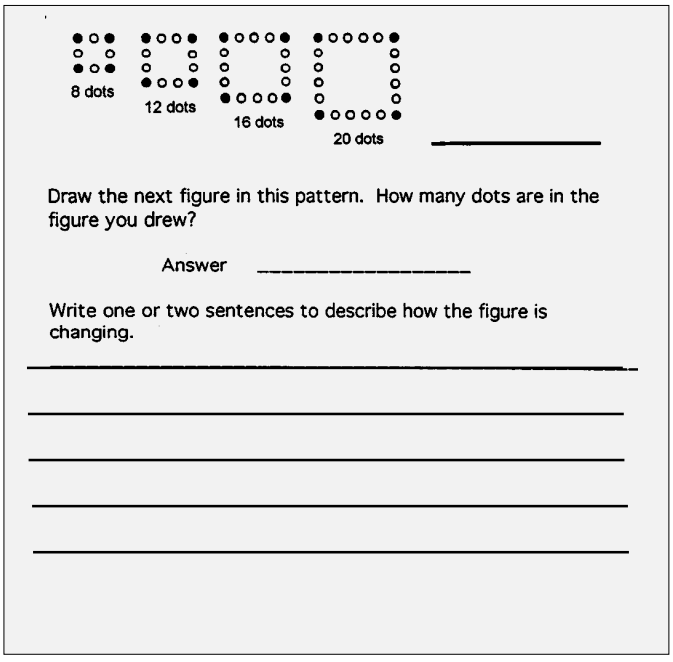
Students:

- **recognize, describe, extend, and create a wide variety of patterns.**
- **represent and describe mathematical relationships.**
- **explore and express relationships using variables and open sentences.**
- **solve for an unknown using manipulative materials.**
- **use a variety of manipulative materials and technologies to explore patterns.**
- **interpret graphs.**
- **explore and develop relationships among two- and three-dimensional geometric shapes.**
- **discover patterns in nature, art, music, and literature.**

This is evident, for example, when students:

- ▲ represent three more than a number is equal to nine as n + 3 = 9.
- ▲ draw leaves, simple wallpaper patterns, or write number sequences to illustrate recurring patterns.
- ▲ write generalizations or conclusions from display data in charts or graphs.

Sample Problem



Standard 3—Mathematics
Intermediate

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:
- **apply a variety of reasoning strategies.**
- **make and evaluate conjectures and arguments using appropriate language.**
- **make conclusions based on inductive reasoning.**
- **justify conclusions involving simple and compound (i.e., and/or) statements.**

This is evident, for example, when students:

▲ use trial and error and work backwards to solve a problem.
▲ identify patterns in a number sequence.
▲ are asked to find numbers that satisfy two conditions, such as $n > -4$ and $n \leq 6$.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:
- **understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded and scientific notation).**
- **understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.**
- **develop an understanding of number theory (primes, factors, and multiples).**
- **recognize order relations for decimals, integers, and rational numbers.**

This is evident, for example, when students:

▲ use prime factors of a group of denominators to determine the least common denominator.
▲ select two pairs from a number of ratios and prove that they are in proportion.
▲ demonstrate the concept that a number can be symbolized by many different numerals as in:

$$\frac{1}{4} = \frac{3}{12} = \frac{25}{100} = 0.25 = 25\%$$

Sample Problems

The table below shows the height of a plant during a period of 3 weeks. Initially the plant was 5 inches tall. The table indicates the growth rate of the plant for week 1 through week 3.

Weeks (W)	0	1	2	3
Height (H) (in inches)	5	8	11	14

A) Write an equation that expresses the height (H) of the plant in terms of the number of weeks (W).

Answer: _____

B) Use the table or your equation to predict the height of the plant after 10 weeks.

2. An inspector found 5 defective cassettes out of a random sample of 200 cassette tapes. If 4,000 cassette tapes are produced each day, how many tapes would you expect to be defective? Write a proportion that can be used to solve this problem and then solve the problem.

Answer: _____

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Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **add, subtract, multiply, and divide fractions, decimals, and integers.**
- **explore and use the operations dealing with roots and powers.**
- **use grouping symbols (parentheses) to clarify the intended order of operations.**
- **apply the associative, commutative, distributive, inverse, and identity properties.**
- **demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).**
- **develop appropriate proficiency with facts and algorithms.**
- **apply concepts of ratio and proportion to solve problems.**

This is evident, for example, when students:

▲ create area models to help in understanding fractions, decimals, and percents.

▲ find the missing number in a proportion in which three of the numbers are known, and letters are used as place holders.

▲ arrange a set of fractions in order, from the smallest to the largest:

$$\frac{3}{4}, \frac{1}{5}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}$$

▲ illustrate the distributive property for multiplication over addition, such as

$2(a + 3) = 2a + 6$.

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **visualize, represent, and transform two- and three-dimensional shapes.**
- **use maps and scale drawings to represent real objects or places.**
- **use the coordinate plane to explore geometric ideas.**
- **represent numerical relationships in one- and two-dimensional graphs.**
- **use variables to represent relationships.**
- **use concrete materials and diagrams to describe the operation of real world processes and systems.**
- **develop and explore models that do and do not rely on chance.**
- **investigate both two- and three-dimensional transformations.**
- **use appropriate tools to construct and verify geometric relationships.**
- **develop procedures for basic geometric constructions.**

This is evident, for example, when students:

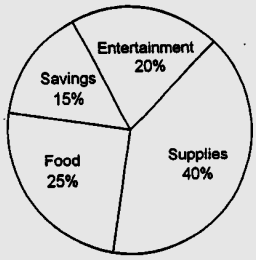
▲ build a city skyline to demonstrate skill in linear measurements, scale drawing, ratio, fractions, angles, and geometric shapes.

▲ bisect an angle using a straight edge and compass.

▲ draw a complex of geometric figures to illustrate that the intersection of a plane and a sphere is a circle or point.

Sample Problems

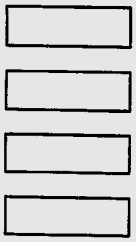
. The graph below shows how Sue spent her allowance last week.



If Sue's allowance is $6.00, how much of her allowance did she spend on entertainment last week?

TASK: SHARING

5. Six students were given four candy bars of equal size. Show how they could divide the candy bars so that each of them received the same amount of candy. Then use the numbers to express how much of a candy bar each student received.



Standard 3—Mathematics

Intermediate

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **estimate, make, and use measurements in real-world situations.**
- **select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.**
- **develop measurement skills and informally derive and apply formulas in direct measurement activities.**
- **use statistical methods and measures of central tendencies to display, describe, and compare data.**
- **explore and produce graphic representations of data using calculators/computers.**
- **develop critical judgment for the reasonableness of measurement.**

This is evident, for example, when students:

- ▲ use box plots or stem and leaf graphs to display a set of test scores.
- ▲ estimate and measure the surface areas of a set of gift boxes in order to determine how much wrapping paper will be required.
- ▲ explain when to use mean, median, or mode for a group of data.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

- **use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.**
- **use estimation to solve problems for which exact answers are inappropriate.**
- **estimate the probability of events.**
- **use simulation techniques to estimate probabilities.**
- **determine probabilities of independent and mutually exclusive events.**

This is evident, for example, when students:

- ▲ construct spinners to represent random choice of four possible selections.
- ▲ perform probability experiments with independent events (e.g., the probability that the head of a coin will turn up, or that a 6 will appear on a die toss).
- ▲ estimate the number of students who might chose to eat hot dogs at a picnic.

Sample Problems

TASK: Donello's Pizzeria

1. Donello's is considering adding a 12" in diameter "large" pizza to its menu. One customer says that adding the large size pizza is unnecessary because it is the same amount of pizza as 2 of the 6" size pizzas. Use mathematics to determine if the customer is correct. Show your work and write a few sentences to explain your answer.

Answer _____

_____

_____

_____

_____

_____

TASK: PAY PLANS

You have just gotten an after school job at City Outfitters. This company offers two different payment plans to its sales employees.

Plan A Earnings:	$110 per week plus 10% of sales
Plan B Earnings:	$80 per week plus 15% of sales

You need to decide which plan to choose and explain why you made this choice.

28. To help you decide, you ask the sales manager what the average weekly sales are. She tells you sales vary a lot, but average around $350 a week. How much would you expect to earn under each payment plan during an average week?

Answer: Plan A _____

Plan B _____

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Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **recognize, describe, and generalize a wide variety of patterns and functions.**
- **describe and represent patterns and functional relationships using tables, charts and graphs, algebraic expressions, rules, and verbal descriptions.**
- **develop methods to solve basic linear and quadratic equations.**
- **develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.**
- **verify results of substituting variables.**
- **apply the concept of similarity in relevant situations.**
- **use properties of polygons to classify them.**
- **explore relationships involving points, lines, angles, and planes.**
- **develop and apply the Pythagorean principle in the solution of problems.**
- **explore and develop basic concepts of right triangle trigonometry.**
- **use patterns and functions to represent and solve problems.**

This is evident, for example, when students:

▲ find the height of a building when a 20-foot ladder reaches the top of the building when its base is 12 feet away from the structure.

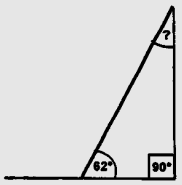
▲ investigate number patterns through palindromes (pick a 2-digit number, reverse it and add the two—repeat the process until a palindrome appears)

	42	86
	+24	+68
palindrome →	66	154
		+451
		605
		+506
	palindrome →	1111

▲ solve linear equations, such as 2(x + 3) = x + 5 by several methods.

Sample Problem

. A painter leaned a ladder up against the wall of my kitchen. The ladder forms an angle of 62° with the floor. What is the measure of the angle formed between the top of the ladder and the wall?



Answer: _____

Standard 3—Mathematics

Commencement

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **construct simple logical arguments.**
- **follow and judge the validity of logical arguments.**
- **use symbolic logic in the construction of valid arguments.**
- **construct proofs based on deductive reasoning.**

This is evident, for example, when students:

▲ prove that an altitude of an isosceles triangle, drawn to the base, is perpendicular to that base.

▲ determine whether or not a given logical sentence is a tautology.

▲ show that the triangle having vertex coordinates of (0,6), (0,0), and (5,0) is a right triangle.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **understand and use rational and irrational numbers.**
- **recognize the order of the real numbers.**
- **apply the properties of the real numbers to various subsets of numbers.**

This is evident, for example, when students:

▲ determine from the discriminate of a quadratic equation whether the roots are rational or irrational.

▲ give rational approximations of irrational numbers to a specific degree of accuracy.

▲ determine for which value of x the expression $\frac{2x + 6}{x - 7}$ is undefined.

Sample Problems

33 Given the true statements:

$\sim a \vee \sim b$

b

$c \rightarrow a$

Which statement is also true?

(1) c (3) $\sim c$

(2) $\sim b$ (4) a

34 Which statement is logically equivalent to the statement: "If you are not part of the solution, then you are part of the problem"?

(1) If you are part of the solution, then you are not part of the problem.

(2) If you are not part of the problem, then you are part of the solution.

(3) If you are part of the problem, then you are not part of the solution.

(4) If you are not part of the problem, then you are not part of the solution.

Key ideas are identified by numbers (1).
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Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.**
- **develop an understanding of and use the composition of functions and transformations.**
- **explore and use negative exponents on integers and algebraic expressions.**
- **use field properties to justify mathematical procedures.**
- **use transformations on figures and functions in the coordinate plane.**

This is evident, for example, when students:

- ▲ determine the coordinates of triangle A(2,5), B(9,8), and C(3,6) after a translation $(x,y) \rightarrow (x + 3, y - 1)$.
- ▲ evaluate the binary operation defined as $x * y = x^2 + (y + x)^2$ for $3 * 4$.
- ▲ identify the field properties used in solving the equation $2(x - 5) + 3 = x + 7$.

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.**
- **manipulate symbolic representations to explore concepts at an abstract level.**
- **choose appropriate representations to facilitate the solving of a problem.**
- **use learning technologies to make and verify geometric conjectures .**
- **justify the procedures for basic geometric constructions.**
- **investigate transformations in the coordinate plane.**
- **develop meaning for basic conic sections.**
- **develop and apply the concept of basic loci to compound loci.**
- **use graphing utilities to create and explore geometric and algebraic models.**
- **model real-world problems with systems of equations and inequalities.**

This is evident, for example, when students:

- ▲ determine the locus of points equidistant from two parallel lines.
- ▲ explain why the basic construction of bisecting a line is valid.
- ▲ describe the various conics produced when the equation $ax^2 + by^2 = c^2$ is graphed for various values of a, b, and c.

Sample Problems

36 *a* On graph paper, draw the graph of the equation $y = x^2 - 4x + 3$, including all values of x in the interval $-1 \leq x \leq 5$. [4]

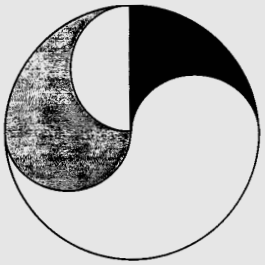
b On the same set of axes, draw the graph of the image of the graph drawn in part *a* after the translation which moves (x,y) to $(x + 3, y + 2)$, and label this graph *b*. [3]

c On the same set of axes, draw the graph of the image of the graph drawn in part *b* after a reflection in the x-axis, and label this graph *c*. [3]

Semicircles

The figure below is made of three small semicircles, all of the same size, and one large circle.

The *diameters* of the semicircles are the same length as the *radius* of the large circle.



Assume that the radius of the large circle is 4 cm long. What is the area of the gray region?

Describe your method: how did you figure it out?

Standard 3—Mathematics

Commencement

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **derive and apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts.**
- **choose the appropriate tools for measurement.**
- **use dimensional analysis techniques.**
- **use statistical methods including measures of central tendency to describe and compare data.**
- **use trigonometry as a method to measure indirectly.**
- **apply proportions to scale drawings, computer-assisted design blueprints, and direct variation in order to compute indirect measurements.**
- **relate absolute value, distance between two points, and the slope of a line to the coordinate plane.**
- **understand error in measurement and its consequence on subsequent calculations.**
- **use geometric relationships in relevant measurement problems involving geometric concepts.**

This is evident, for example, when students:

- ▲ change mph to ft/sec.
- ▲ use the tangent ratio to determine the height of a tree.
- ▲ determine the distance between two points in the coordinate plane.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

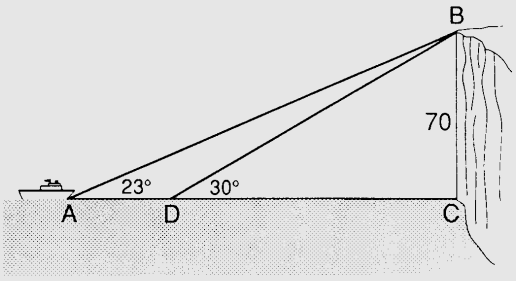
- **judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics.**
- **judge the reasonableness of a graph produced by a calculator or computer.**
- **use experimental or theoretical probability to represent and solve problems involving uncertainty.**
- **use the concept of random variable in computing probabilities.**
- **determine probabilities using permutations and combinations.**

This is evident, for example, when students:

- ▲ construct a tree diagram or sample space for a compound event.
- ▲ calculate the probability of winning the New York State Lottery.
- ▲ develop simulations for probability problems for which they do not have theoretical solutions.

Sample Problems

39 As shown in the accompanying diagram, a ship is headed directly toward a coastline formed by a vertical cliff $\overline{BC}$, 70 meters high. At point A, the angle of elevation from the ship to B, the top of the cliff, is 23°. A few minutes later at point D, the angle of elevation increased to 30°.



a To the *nearest meter*, find:
(1) DC [3]
(2) AC [3]
(3) AB [3]

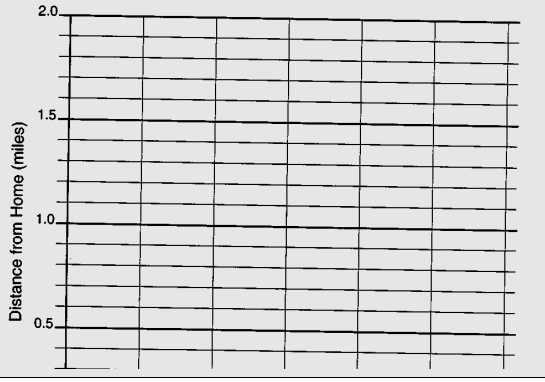
b To the *nearest meter*, what is the distance between the ship's position at the two sightings?

Every morning Walker Bryce walks 1.7 miles to school.

He leaves his house at 8:05 and walks 1.2 miles, then he waits for Bobby and Denise.

When they show up, all three of them start walking to school together. They arri ten minutes later at 8:55.

Draw a graph that could show Walker's journey to school.



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Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **use function vocabulary and notation.**
- **represent and analyze functions using verbal descriptions, tables, equations, and graphs.**
- **translate among the verbal descriptions, tables, equations and graphic forms of functions.**
- **analyze the effect of parametric changes on the graphs of functions.**
- **apply linear, exponential, and quadratic functions in the solution of problems.**
- **apply and interpret transformations to functions.**
- **model real-world situations with the appropriate function.**
- **apply axiomatic structure to algebra and geometry.**
- **use computers and graphing calculators to analyze mathematical phenomena.**

This is evident, for example, when students:

- ▲ determine, in more than one way, whether or not a specific relation is a function.
- ▲ explain the relationship between the roots of a quadratic equation and the intercepts of its corresponding graph.
- ▲ use transformations to determine the inverse of a function.

Sample Problem

Fibonacci Pattern

This is the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, …

Each number (starting with the "2") is the sum of the previous two. For example,

1 + 1 = 2 and 2 + 3 = 5.

The number that comes after 21, in the above sequence, is 34 because:

13 + 21 = 34.

Now look at the pattern of odd and even numbers in this sequence. If we replace each odd number with "O" and each even with "E," we get:

O, O, E, O, O, E, O, O, …

Only one of the following statements is correct.

Decide which one you think is correct and explain in detail your choice.

A. The pattern, O, O, E, does NOT repeat forever.

B. The pattern, O, O, E, repeats forever.

Standard 3—Mathematics

Four-year sequence in mathematics

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **construct indirect proofs or proofs using mathematical induction.**
- **investigate and compare the axiomatic structures of various geometries.**

This is evident, for example, when students:

- ▲ prove indirectly that: if n^2 is even, n is even.
- ▲ prove using mathematical induction that: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
- ▲ explain the axiomatic differences between plane and spherical geometries.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **understand the concept of infinity.**
- **drecognize the hierarchy of the complex number system.**
- **model the structure of the complex number system.**
- **recognize when to use and how to apply the field properties.**

This is evident, for example, when students:

- ▲ relate the concept of infinity when graphing the tangent function.
- ▲ show that the set of complex numbers form a field under the operations of addition and multiplication.
- ▲ show that the set of complex numbers forms a field under the operations of addition and multiplication.
- ▲ represent a complex number in polar form.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

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Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **use appropriate techniques, including graphing utilities, to perform basic operations on matrices.**
- **use rational exponents on real numbers and all operations on complex numbers.**
- **combine functions using the basic operations and the composition of two functions.**

This is evident, for example, when students:

- ▲ relate specific matrices to certain types of transformations of points on the coordinate plane.
- ▲ evaluate expressions with fractional exponents, such as $8^{2/3} 4^{-1/2}$.
- ▲ determine the value of compound functions such as (f o g) (x).

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **model vector quantities both algebraically and geometrically.**
- **represent graphically the sum and difference of two complex numbers.**
- **model and solve problems that involve absolute value, vectors, and matrices.**
- **model quadratic inequalities both algebraically and graphically.**
- **model the composition of transformations.**
- **determine the effects of changing parameters of the graphs of functions.**
- **use polynomial, rational, trigonometric, and exponential functions to model real-world relationships.**
- **use algebraic relationships to analyze the conic sections.**
- **use circular functions to study and model periodic real-world phenomena.**
- **illustrate spatial relationships using perspective, projections, and maps.**
- **represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations.**
- **analyze spatial relationships using the Cartesian coordinate system in three dimensions.**

This is evident, for example, when students:

- ▲ determine coordinates which lie in the solution of the quadriatic inequality, such as $y < x^2 + 4x + 2$.
- ▲ find the distance between two points in a three-dimension coordinate system.
- ▲ describe what happens to the graph when b increases in the function $y = x^2 + bx + c$.

Standard 3—Mathematics

Four-year sequence in mathematics

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **derive and apply formulas relating angle measure and arc degree measure in a circle.**
- **prove and apply theorems related to lengths of segments in a circle.**
- **define the trigonometric functions in terms of the unit circle.**
- **relate trigonometric relationships to the area of a triangle and to the general solutions of triangles.**
- **apply the normal curve and its properties to familiar contexts.**
- **design a statistical experiment to study a problem and communicate the outcomes, including dispersion.**
- **use statistical methods, including scatter plots and lines of best fit, to make predictions.**
- **apply the conceptual foundation of limits, infinite sequences and series, the area under a curve, rate of change, inverse variation, and the slope of a tangent line to authentic problems in mathematics and other disciplines.**
- **determine optimization points on a graph.**
- **use derivatives to find maximum, minimum, and inflection points of a function.**

This is evident, for example, when students:

▲ use a chi-square test to determine if one cola really tastes better than another cola.

▲ can illustrate the various line segments which represent the sine, cosine, and tangent of a given angle on the unit circle.

▲ calculate the first derivative of a function using the limit definition.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

- **interpret probabilities in real-world situations.**
- **use a Bernoulli experiment to determine probabilities for experiments with exactly two outcomes.**
- **use curve fitting to predict from data.**
- **apply the concept of random variable to generate and interpret probability distributions.**
- **create and interpret applications of discrete and continuous probability distributions.**
- **make predictions based on interpolations and extrapolations from data.**
- **obtain confidence intervals and test hypotheses using appropriate statistical methods.**
- **approximate the roots of polynomial equations.**

This is evident, for example, when students:

▲ verify the probabilities listed for the state lottery for second, third, and fourth prize.

▲ use graphing calculators to generate a curve of best fit for an array of data using linear regression.

▲ determine the probability of getting at least 3 heads on 6 flips of a fair coin.

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Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **solve equations with complex roots using a variety of algebraic and graphical methods with appropriate tools.**
- **understand and apply the relationship between the rectangular form and the polar form of a complex number.**
- **evaluate and form the composition of functions.**
- **use the definition of a derivative to examine the properties of a function.**
- **solve equations involving fractions, absolute values, and radicals.**
- **use basic transformations to demonstrate similarity and congruence of figures.**
- **identify and differentiate between direct and indirect isometries.**
- **analyze inverse functions using transformations.**
- **apply the ideas of symmetries in sketching and analyzing graphs of functions.**
- **use the normal curve to answer questions about data.**
- **develop methods to solve trigonometric equations and verify trigonometric functions.**
- **describe patterns produced by processes of geometric change, formally connecting iteration, approximations, limits, and fractals.**
- **extend patterns and compute the nth term in numerical and geometric sequences.**
- **use the limiting process to analyze infinite sequences and series.**
- **use algebraic and geometric iteration to explore patterns and solve problems.**
- **solve optimization problems.**
- **use linear programming and difference equations in the solution of problems.**

This is evident, for example, when students:

▲ transform polar coordinates into rectangular forms.

▲ find the maximum height of an object projects upward with a given initial velocity.

▲ find the limit of expressions like $\frac{n-2}{3n+5}$ as n goes to infinity.